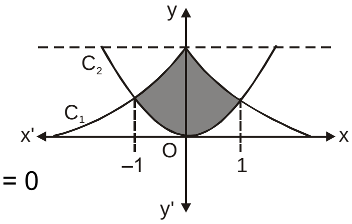
EXERCISE – II

HINTS & SOLUTIONS

Sol.1 **A,B**

Point of intersection of two curves C_1 and C_2

$$\frac{1}{1+x^2} = \frac{x^2}{2}$$



$x^4 + x^2 – 2 = 0$

$(x^2 + 2)(x^2 – 1) = 0$

$\mathbf{x = \pm 1}$

Area bounded by the curve C_1 and $y = 0$ is

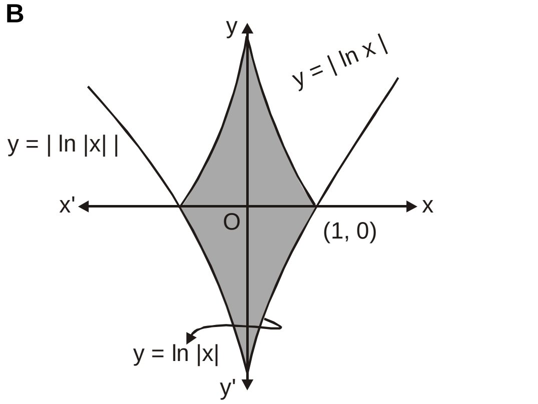
$$= 2\int_0^{\infty}\frac{1}{1+x^2}dx = 2\left[\tan^{-1}x\right]_0^{\infty} = 2\left[\frac{\pi}{2}\right] = \pi$$

Area bounded by

C_1 and C_2 is $2\left(\int_0^1 \frac{1}{1+x^2}dx - \int_0^1 \frac{x^2}{2}dx\right)$

$$= 2\left[\tan^{-1}x\right]_0^1 - \frac{1}{3}\left[x^3\right]_0^1 = \frac{\pi}{3} - \frac{1}{3}$$ **Ans.**

Sol.2 **B**

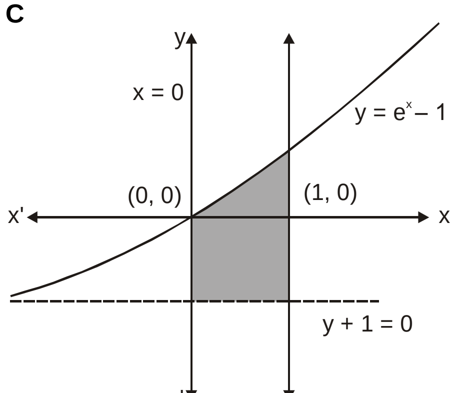


Area enclosed by the curves y = ln x, y = ln |x|

y=|ln x| and y= | ln |x| | is $4\int_0^1 |\ln x|\,dx$

$$= 4\left[x \ln x - x\right]_0^1 = 4$$

Sol.3 **C**



According to questions f''(x) = f'(x)

$$\Rightarrow \int f''(x)dx = \int f'(x)dx$$

$f'(x) = f(x) + 4 \Rightarrow f'(0) = f(0) + 4 \Rightarrow 4 = 1$

Now $f'(x) = f(x) + 1$

$$\Rightarrow \frac{f'(x)}{f(x)+1} = 1 \Rightarrow \int \frac{f'(x)}{f(x)+1}dx = \int dx$$

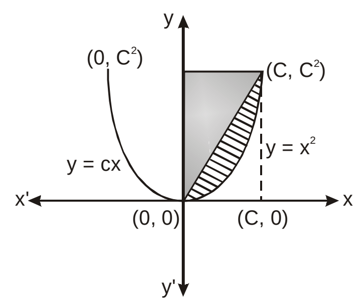
$\ln|f(x) + 1| = x + C_2 \Rightarrow \ln|f(0) + 1| = 0 + C_2 \Rightarrow C_2 =$

$\ln| f(x) + 1 = x \Rightarrow f(x) + 1 = e^x \Rightarrow f(x) = e^x – 1$

$$A = \int_0^1 (e^x - 1)dx + 1\times 1 \Rightarrow A = e – 2 + 1 \Rightarrow A = e – 1$$

Sol.4 **A,C**

$$\text{Area (T)} = \frac{1}{2}\times C^2 \times C = \frac{C^3}{2}$$



$$\text{Area (R)} = \int_0^C (Cx - x^2)dx = \frac{C^3}{6}$$

$$\lim_{C\to 0^+}\frac{\text{Area(T)}}{\text{Area(R)}} = \lim_{C\to 0^+}\frac{6C^3}{2\times C^3} = 3$$

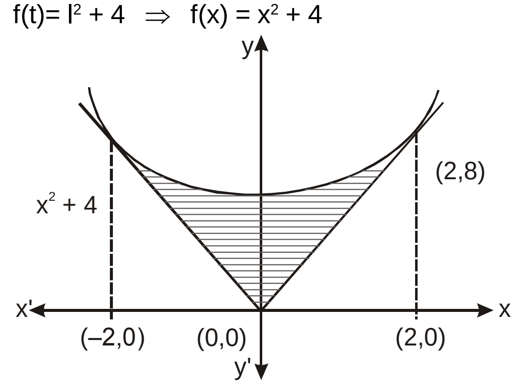
Sol.5 **B**

$h(x) = f\{g(x)\} \Rightarrow h(x) = f(2x + 1)$

$4x^2 + 4x + 5 = f(2x + 1)$

$\Rightarrow f(2x + 1) = (2x + 1)^2 + 4$

$f(t)= l^2 + 4 \Rightarrow f(x) = x^2 + 4$



Let the pair of S.l assing through the origin is y = mx and tangent to $y = x^2 + 4$

$\therefore\ x^2 + 4 = mx \quad \Rightarrow x^2 = mx + 4 = 0 \quad \Rightarrow\ D = 0$

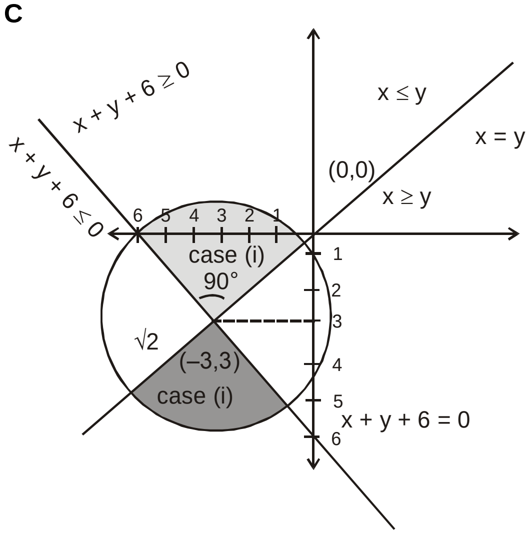
$\Rightarrow m^2 - 16 = 0 \quad \Rightarrow m = \pm 4 \quad \Rightarrow\ x \tan y = \pm 4x$

Point of untense of $y = 4x$ and $y = x^2 + 4$

$x^2 + 4 = 4x \Rightarrow (x - 2)^2 = 0 \Rightarrow x = 2$

$$A = 2\int_0^2 (x^2 + 4)dx - 2 \times \frac{1}{2} \times 2 \times 8 \ \Rightarrow\ A = \frac{16}{3}$$

Sol.6 C



$f(x) = x^2 + 6x + 1 \Rightarrow f(y) = y^2 + 6y + 1$

$f(x) + f(y) = x^2 + y^2 + 6x + 6y^{+2} \le 0$ snous interior

area of circle radius of circle is = 4

$f(x) - f(y) = x^2 - y^2 + 6x - 6y \le 0$

$\Rightarrow (x - y)\ (x + y + 6) \le 0$

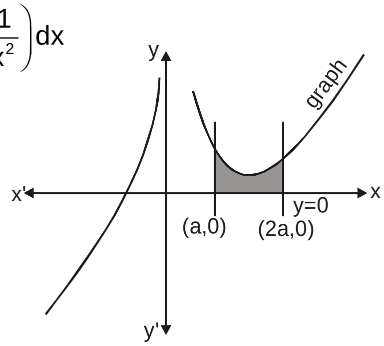
Case (i) $x - y \ge 0$ and $x + y + 6 \le 0$

or Case (ii) $x - y \le 0$ and $x^2 + y + 6 \ge 0$,

$$A = \frac{1}{2} \times 16 \times \frac{\pi}{2} \times 2 \quad \Rightarrow \quad A = 8\pi$$

Sol.7 D

$$A = \int_a^{2a} \left(\frac{x}{6} + \frac{1}{x^2}\right)dx$$



$$A = \left[\frac{x^2}{12} + -\frac{1}{x}\right]_a^{2a}$$

$$A = \frac{a^2}{4} + \frac{1}{2a}$$

Now, $\dfrac{dA}{da} = \dfrac{a}{2} - \dfrac{1}{2a^2}$

For maximum and minimum value of A

$$\frac{dA}{da} = 0 \Rightarrow \frac{a}{2} = \frac{1}{2a^2} \Rightarrow a^3 = 1$$

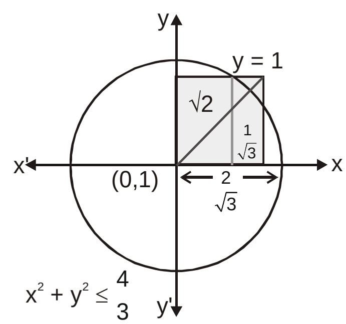
$$\therefore\ \frac{d^2A}{da^2} = \frac{1}{2} + \frac{1}{a^3}$$

$$\left.\frac{d^2A}{da^2}\right)_{a=1} = \frac{1}{2} + 1 > 0 \Rightarrow \text{A is least value at } a = 1$$

Sol.8 D

$$\frac{2}{\sqrt{3}} = \frac{2}{1.732} = 1.15$$

$$x^2 + 1 = \frac{4}{3}$$



$$x^2 = \frac{1}{3} \Rightarrow x = \frac{1}{\sqrt{3}}$$

$$R_1\pi R_2 = \frac{1}{\sqrt{3}} \times 1 + \int_{\frac{1}{\sqrt{3}}}^{1} \sqrt{\frac{4}{3} - x^2}\,dx$$

$$R_1\pi R_2 = \frac{1}{\sqrt{3}} + \left[\frac{x}{2}\sqrt{\frac{4}{3} - x^2} + \frac{2}{3}\sin^{-1}\left(\frac{x\sqrt{3}}{2}\right)\right]_{\frac{1}{\sqrt{3}}}^{1}$$

$$R_1\pi R_2 = \frac{1}{\sqrt{3}} + \left[\frac{x}{2}\sqrt{\frac{4}{3} - x^2} + \frac{2}{3}\sin^{-1}\left(\frac{x\sqrt{3}}{2}\right)\right]_{\frac{1}{\sqrt{3}}}^{1}$$

$$= \frac{1}{2\sqrt{3}} + \frac{2\pi}{9} - \frac{\pi}{9} = \frac{1}{2\sqrt{3}} + \frac{\pi}{6} = \frac{1}{\sqrt{3}} + \frac{\pi}{9}$$

$$= \frac{3\sqrt{3} + \pi}{9} = \frac{9}{\sqrt{3}} = 3$$

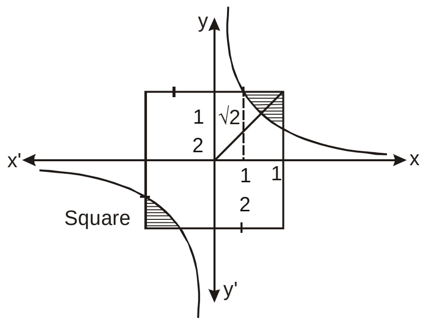
Sol.9 A,D

$| x | \le 1$ $| y | \le 1$

$\Rightarrow\ -1 \le x \le 1$

& $1 \le y \le 1$

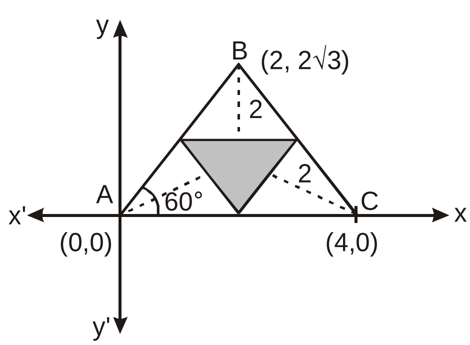
& $xy \le \dfrac{1}{2} \quad \Rightarrow\ y \le \dfrac{1}{2x}$



$$\text{Required Area} = 2\int_{\frac{1}{2}}^{1}\left(1-\frac{1}{2x}\right)dx$$

$$= 2\left(x-\frac{1}{2}\tan x\right)_{\frac{1}{2}}^{1} = 1 - \ln 2 \text{ sq. unit}$$

Sol.10 B

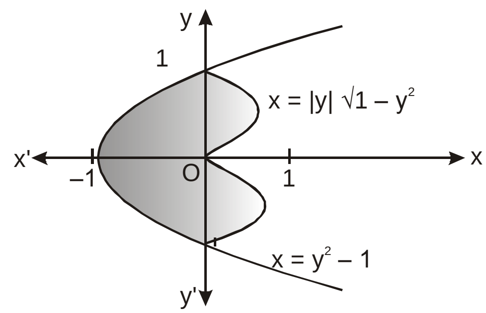


Area bounded by the curve traced by P is

$$= \frac{\sqrt{3}}{4}\times(4)^2 - 3\times\frac{1}{2}\times 4\times\frac{\pi}{3}$$

$$= 4\sqrt{3} - 2 \text{ sq. unit.}$$

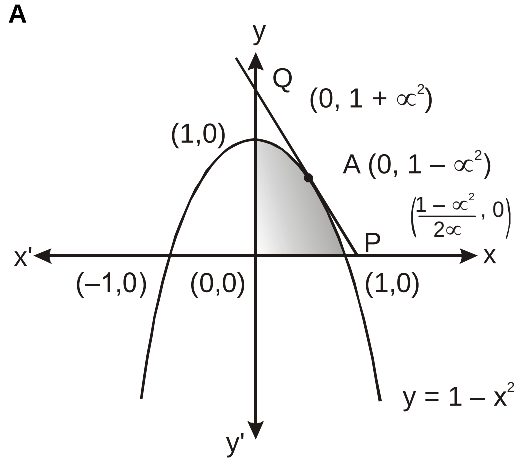
Sol.11 D



$$\text{Area} = 2\int_0^1 y\sqrt{1-y^2}\,dy + 2\int_0^1 (y^2-1)dy$$

$\Rightarrow$ A = 2 **Ans.**

Sol.12 A



Example of tangent at A($\propto$, 1 – $\propto^2$) for the curve $y = 1 - x^2$ is $y + 1 - \propto^2 = 2 - 2\propto x$

$$2\propto x + y = 1 + \propto^2 \qquad \Rightarrow \frac{x}{\frac{1+\propto^2}{2\propto}} + \frac{y}{1+\propto^2} = 1$$

$$\text{Area of } \Delta OPQ = \frac{1}{2}\times\frac{(1+\propto^2)^2}{2\propto} \Rightarrow \Delta = \frac{(1+\propto^2)^2}{4\propto}$$

$$\frac{d\Delta}{d\propto} = \frac{(\propto^2+1)(3\propto^2-1)}{\propto} \text{ for minimum value of } \Delta$$

$$\frac{d\Delta}{d\propto} = 0 \Rightarrow \frac{(3\propto^2-1)(\propto^2+1)}{\propto} = 0 \Rightarrow \propto = \frac{1}{\sqrt{3}}$$

$$\text{Minimum Area of } \Delta = \frac{4\sqrt{3}}{9}$$

According to questions $\Rightarrow \sqrt{4+12} = 4$